

Perturbative computations in string field theory

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Abstract

These notes describe how perturbative on-shell and off-shell string amplitudes can be computed using string field theory. Computational methods for approximating arbitrary amplitudes are discussed, and compared with standard world-sheet methods for computing on-shell amplitudes. These lecture notes are not self-contained; they contain the material from W. Taylor's TASI 2003 lectures not covered in the recently published "TASI 2001" notes hep-th/0311017 by Taylor and Zwiebach, and should be read as a supplement to those notes.

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1 Introduction

String field theory is a nonperturbative, off-shell formulation of string theory in target space. Over the last several years, nonperturbative solutions of string field theory have been identified and studied in detail. These solutions represent nonperturbative shifts in the string background. The fact that string field theory admits such solutions is an important manifestation of the background independence of the theory. Developing a completely background-independent formulation of string theory, where different string backgrounds arise on equal footing as solutions of a system of equations expressed in terms of a common set of degrees of freedom is probably necessary in order to have a sensible description of cosmology and early-universe physics in terms of string theory. An introduction to string field theory and recent developments in this area is given in the “TASI 2001” lecture notes by Taylor and Zwiebach [1]; those lecture notes give a detailed introduction to Witten’s cubic string field theory and describe nonperturbative results on tachyon condensation in this theory.

While the primary motivation for string field theory is to develop tools for understanding nonperturbative phenomena in string theory, recent developments have also shown that string field theory can be useful in performing perturbative off-shell and on-shell computations in string theory. These lecture notes give a short introduction to such calculations. They should be read as a supplement to [1]. Throughout these notes we use the notation and conventions of [1]. We assume that the reader is familiar in particular with the material in sections 4 and 6 of [1], where the basic structure of Witten’s OSFT is explained in detail. The content of these notes formed part of the material for lectures by W. Taylor at the TASI 2003 summer school; the remainder of the material discussed in those lectures is described in [1].

The fact that Witten’s cubic bosonic open string field theory [2] reproduces all on-shell perturbative string amplitudes was demonstrated many years ago [3, 4, 5]. In perturbative string theory, a general on-shell N -point amplitude is computed by performing an integral of the form

$$\int d\mu \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_\mu,$$

where \mathcal{O}_i are a set of vertex operators on the string world sheet, and the integral $\int d\mu$ is taken over the moduli space of Riemann surfaces with n marked points, summed over topology. In general, the modular integral becomes progressively more difficult to perform explicitly as the topology of the Riemann surface becomes more complicated. Recent computations of genus 2 amplitudes in superstring theory are described in [6]; analogous amplitudes for genus 3 or higher have not yet been computed.

A nice feature of string field theory is that the integration over moduli space is carried out in a natural and simple fashion. In Feynman-Siegel gauge, the SFT propagator is

$$\frac{1}{L_0} = \int_0^\infty dt e^{-tL_0}. \quad (1)$$

Here, the Schwinger parameter t can be interpreted in the string world-sheet language as a modular parameter describing the length of a propagating strip. Thus, in SFT the modular

integral simply becomes an integration over a set of Schwinger parameters, which is quite straightforward to organize. For example, consider the planar one-loop 3-point function for open strings. The world-sheet in this case has the form of an annulus with three marked points on the boundary. The usual parameterization of the world-sheet moduli space would be in terms of the ratio of radii r_2/r_1 for the annulus and the angles θ, ϕ between the marked points. In SFT, the modular parameters are simply the lengths $t_i, i = 1, 2, 3$ of the three strips connecting three 3-point vertices connected in a circle, each with an external edge. While the modular integrals in these two coordinatizations can be related through a conformal map, the explicit relationship between the Witten SFT parameterization and the usual CFT parameterization becomes quite complicated for surfaces of higher topology. It was shown in [3, 4, 5] that in Feynman-Siegel gauge, the set of SFT diagrams with the propagator (1) precisely reproduces all on-shell open string amplitudes by covering the moduli space of Riemann surfaces with the correct measure. This guarantees that general perturbative calculations in SFT will reproduce the usual open string scattering amplitudes when restricted to on-shell external momenta. An example of this correspondence is the tree level 4-point Veneziano amplitude, which was shown by Giddings [7] to be correctly reproduced by Witten's SFT; we will consider this example in more detail in Section 3.

While on-shell amplitudes are invariant under arbitrary conformal transformations of the string world-sheet, the same is not true of off-shell amplitudes. Generally, a different choice of conformal coordinates will give a different definition for off-shell amplitudes. Essentially, string field theory provides a consistent decomposition of the Riemann surface for each string diagram, giving a systematic off-shell extension of string theory. The goal of these lectures is to describe methods for computing off-shell amplitudes using Witten's open string field theory.

2 Computing off-shell amplitudes

There are a number of different ways of using OSFT to compute off-shell amplitudes. We briefly summarize four of these approaches here:

a) Naive field theory approach:

In this approach, we treat SFT as a standard field theory, albeit with an infinite number of space-time fields. We evaluate the amplitude by summing over all fields which may arise on each internal propagator. For example, the tree level 4-point amplitude for the tachyon would be written abstractly as

$$\sum_I V_{I\phi_1\phi_2} \frac{1}{L_0(I)} V_{I\phi_3\phi_4} , \quad (2)$$

where I is the internal field, and ϕ_i are the external tachyon fields. Such computations cannot currently be performed exactly, except in the simplest cases¹. It is possible,

¹for an example of an exact computation of this type, see [8], where the off-shell zero-momentum 4-point function of the massless gauge field is computed exactly

however, to approximate such a calculation by truncating the sum over intermediate fields at a fixed mass level L . We will refer to such an approximation as “level truncation on fields”, to distinguish it from a related method described below involving level truncation on oscillators. Because the number of fields grows exponentially in L , the complexity of this method grows exponentially in L . This method was used in both earlier [9] and more recent work [10, 11] on the tachyon condensation problem in OSFT to compute coefficients in the tachyon effective potential.

b) Conformal mapping:

Another approach is to explicitly construct a conformal mapping which takes the Witten parameterization of moduli space to a parameterization more convenient for standard conformal field theory calculations. This is the method first used by Giddings [7] to compute the Veneziano amplitude in Witten’s OSFT. Following Giddings’ calculation, this method was used to compute the off-shell generalization of the Veneziano amplitude [12, 13], the off-shell 4-point function for general fields [14], the off-shell Koba-Nielsen (tree level N-point) amplitude [15], and the off-shell one-loop N-point amplitude [16]. Finally, Samuel showed [17] that an arbitrary loop amplitude could be computed using this approach. This approach has the advantage that it gives, in principle, analytic results. These analytic results are often very complicated, however, as the conformal mapping can be quite nontrivial even in simple cases.

c) Oscillator method:

The key observation underlying the oscillator method is the fact that both the cubic string vertex and the propagator can be expressed in terms of exponentials of quadratic forms in the oscillator raising and lowering operators. Given a particular diagram topology, with v cubic vertices and e internal edges, the set of cubic vertices describes a squeezed state

$$\langle V| = \langle V_3|_{123} \otimes \langle V_3|_{456} \otimes \cdots \otimes \langle V_3|_{(3v-2)(3v-1)(3v)} \in (\mathcal{H}^*)^{3v}. \quad (3)$$

(The 2- and 3-vertices $|V_2\rangle, |V_3\rangle$ are described in detail in section 6 of [1].) The e internal edges connecting the labeled edges $i_k, j_k \leq 3v, k \leq e$, can be described by taking a squeezed state

$$|D\rangle = |V_2\rangle_{i_1 j_1} \otimes |V_2\rangle_{i_2 j_2} \otimes \cdots \otimes |V_2\rangle_{i_e j_e}, \quad (4)$$

describing the connection between the edges, and inserting a propagator

$$P = \int \prod_{k=1}^e dT_k e^{-\frac{1}{2}T_k(L_0^{(i_k)} + L_0^{(j_k)})}. \quad (5)$$

The complete set of amplitudes associated with the diagram of interest is then given by an integral over internal (loop) momenta

$$\mathcal{A} = \int \prod_{i=1}^{1+e-v} d^{26}q_i \langle V|P|D\rangle. \quad (6)$$

This expression gives a state in $(\mathcal{H}^*)^{3v-2e}$, which can be contracted with external string states to get any particular amplitude associated with the relevant diagram. Because the vertices are described by squeezed states, and L_0 is quadratic in oscillator modes, the expression (6) gives an integral over squeezed states; the integrand can be computed using standard squeezed state methods in terms of the infinite-dimensional Neumann matrices V_{nm}^{rs}, X_{nm}^{rs} . Thus, a closed-form expression can be computed for any amplitude. Using current technology, such amplitudes cannot be computed exactly. Truncating the Neumann matrices to finite size by imposing a cutoff on level number leads to expressions in terms of finite-size matrices which can be readily calculated. The resulting amplitudes include contributions from an infinite family of space-time fields, namely those fields associated with string states containing oscillators up to a fixed level L . We refer to this method as “level truncation on oscillators”. This method, unlike method (a), grows polynomially in L . This method was developed for Witten’s OSFT in [18].

- d) Another method for computing OSFT amplitudes which has been developed recently, primarily by Bars, Kishimoto, Matsuo, and Park [19] relies on the Moyal product representation of the star product [20]. The idea here is that the star product can be diagonalized at the expense of complicating the operator L_0 . This method presents an alternative approach to performing calculations which seems to give numerical results comparable to those achieved using level truncation and the usual oscillator representation of the vertices and propagator as in methods (a), (c). For a review of this approach see [21].

In these lectures, we will describe some particular open string amplitudes which have been computed using methods (a) and/or (c). At this point we make some brief comments summarizing some relevant features of the four methods just described.

- Amplitudes computed using method (b) are in principle exact, while methods (a, c, d) are approximate. In all cases, however, numerical approximations are necessary to get concrete numbers for the amplitudes; in general, method (b) leads to complicated expressions in terms of quantities defined implicitly through integral relations, which can only be approximated numerically.
- For finite amplitudes, the methods of field and oscillator level truncation (approaches (a, c)) have been found empirically to give errors which go as $a_1/L + a_2/L^2 + \dots$, where a_n are undetermined constants, and L is the level of truncation. There exists at this time no general proof of this result, but it seems fairly robust, and leads to highly accurate predictions for quantities which are known through other methods.
- For genus $g > 1$, it is difficult to get even approximate results using standard CFT methods. In an impressive paper [17], Samuel showed that in principle any loop amplitude can be computed using method (b); the calculations involved, while rather

complex, are well-defined for amplitudes at any loop order. Methods (a, c) have the advantage that they are not any more complicated conceptually for higher-loop amplitudes than for tree amplitudes, although they are more involved algebraically; it would be straightforward to automate calculations using these methods for diagrams of arbitrary complexity.

- Method (c) is generally useful for accurate computation of n -point amplitudes for small n , while method (a) is better for large n , since (a) scales polynomially in n but exponentially in L while (c) scales polynomially in L but exponentially in n .

3 Example: 4-tachyon amplitude

As a simple example of calculational approaches (a) and (c) we now consider the off-shell 4-tachyon amplitude. We first describe the calculation of this amplitude in some detail at $p = 0$, where it corresponds to the quartic term in the effective potential for the tachyon zero-mode. At the end of this section we briefly describe the generalization of the calculation to nonzero momentum. This calculation is described in greater detail in [18].

Applying the general methodology described in (c) to the 4-tachyon amplitude, we wish to consider the amplitude

$$\mathcal{A} = \frac{1}{2}g^2 \int dt (\langle V_3|_{123} \otimes \langle V_3|_{456}) e^{-t/2(L_0^{(3)} + L_0^{(4)})} (|0\rangle_1 \otimes |0\rangle_2 \otimes |V_2\rangle_{34} \otimes |0\rangle_5 \otimes |0\rangle_6). \quad (7)$$

In oscillator language, this amplitude becomes

$$\begin{aligned} \frac{1}{2}(\kappa g)^2 \int dt e^t \left[\langle 0|_{34} \exp \left(-\frac{1}{2}a_n^{(3)} V_{nm}^{11} a_m^{(3)} - \frac{1}{2}a_n^{(4)} V_{nm}^{11} a_m^{(4)} \right) e^{-N^{(3)}t} \right. \\ \left. \times \exp \left(-(a_n^{(3)})^\dagger C_{nm} (a_m^{(4)})^\dagger \right) |0\rangle_{34} \times \text{ghosts} \right] \end{aligned} \quad (8)$$

where $C_{nm} = (-1)^n \delta_{nm}$, and where we have used $N^{(3)} = N^{(4)}$ for the total matter oscillator number. The ghost part of the expression is practically identical in form, replacing $V \rightarrow X$ and replacing the matter oscillators a_n, a_m by c_n, b_m .

The matter part of the matrix element in (8) can be written as an inner product of two squeezed states

$$I_{\text{matter}} = \langle 0| e^{-\frac{1}{2}a \cdot \tilde{V} \cdot a} e^{-\frac{1}{2}a^\dagger \cdot S(t) \cdot a^\dagger} |0\rangle \quad (9)$$

where

$$\tilde{V} = \begin{pmatrix} V_{nm}^{11} & 0 \\ 0 & V_{nm}^{11} \end{pmatrix} \quad (10)$$

and

$$S(t) = \begin{pmatrix} 0 & \delta_{mn}(-1)^m e^{-mt} \\ \delta_{mn}(-1)^m e^{-mt} & 0 \end{pmatrix} \quad (11)$$

are expressed in terms of blocks, each of which is an infinite-dimensional matrix with indices $n, m \in \mathbf{Z}$.

The matrix generalization [22] of standard formulae for squeezed state inner products gives

$$I_{\text{matter}} = \det \left(\mathbb{1} - S(t) \cdot \tilde{V} \right)^{-13}. \quad (12)$$

A similar result holds for the ghosts. Writing

$$x = e^{-t}, \quad (13)$$

the full amplitude (8) can then be given as

$$\mathcal{A} = \frac{(\kappa g)^2}{2} \int_0^1 \frac{dx}{x^2} \frac{\det \left(\mathbb{1} + S(x) \cdot \tilde{X} \right)}{\det \left(\mathbb{1} - S(x) \cdot \tilde{V} \right)^{13}}. \quad (14)$$

The expression (14) diverges as $x \rightarrow 0$. This divergence arises from the intermediate tachyon state. If we are interested in computing the effective tachyon potential at zero momentum,

$$V_{\text{eff}} = -\frac{1}{2}\phi^2 + \frac{g\kappa}{3}\phi^3 + c_4\left(\frac{g\kappa}{3}\right)^2\phi^4 + \dots, \quad (15)$$

then we wish to drop terms associated with the intermediate tachyon field. Terms in the ratio of determinants appearing in (14) at order x^n correspond to intermediate states in the 4-tachyon tree amplitude at excitation level $L = n$. Thus, dropping the intermediate tachyon term should give

$$c_4 = -\frac{9}{2} \int_0^1 \frac{dx}{x^2} \left[\frac{\det(1 + S \cdot \tilde{X})}{\det(1 - S \cdot \tilde{V})^{13}} - 1 \right]. \quad (16)$$

The result (16) contains infinite matrices; an exact result for c_4 cannot currently be computed analytically from this expression. We can, however, use level truncation on oscillators to give an expression in terms of finite-size matrices which can be computed exactly. For example, let us consider the simplest truncation, namely the truncation to only fields composed of oscillators a_1, b_1 , and c_1 . The matrices S, \tilde{V}, \tilde{X} then become the 2×2 matrices

$$S = \begin{pmatrix} 0 & -x \\ -x & 0 \end{pmatrix}, \quad (17)$$

$$\tilde{V} = \begin{pmatrix} \frac{5}{27} & 0 \\ 0 & \frac{5}{27} \end{pmatrix}, \quad (18)$$

$$\tilde{X} = \begin{pmatrix} -\frac{11}{27} & 0 \\ 0 & -\frac{11}{27} \end{pmatrix}, \quad (19)$$

where we have used $V_{11}^{11} = 5/27, X_{11}^{11} = -11/27$. This gives the resulting formula for the oscillator level 1 approximation to c_4

$$c_4^{(1)} = -\frac{9}{2} \int_0^1 \frac{dx}{x^2} \left[\frac{1 - \frac{121}{729}x^2}{\left(1 - \frac{25}{729}x^2\right)^{13}} - 1 \right]. \quad (20)$$

Expanding the integrand in a power series in x , we have

$$c_4 \approx -\frac{9}{2} \int_0^1 dx \left[\frac{68}{243} + \frac{650}{19683} x^2 + \dots \right]. \quad (21)$$

The first term in the expansion arises from the two intermediate fields at level $L = 2$ ($Ba_{-1} \cdot a_{-1}|0\rangle, \beta b_{-1}c_{-1}|0\rangle$). The second term in the expansion arises from level $L = 4$ fields, etc.. The expression (20) can be integrated exactly, leading to the oscillator level 1 approximation

$$c_4^{(1)} \approx -1.309. \quad (22)$$

Considering only the leading term in (21) amounts to performing a level truncation on fields at level 2, and gives

$$c_4^{[2]} = -34/27 \approx -1.259. \quad (23)$$

If we wish to include all intermediate fields to level 4 (method (a)), we would need to include oscillators up to level 3, and include the first two terms in an expansion analogous to (21). This calculation is easy to carry out, and gives

$$c_4^{[4]} \approx -1.472. \quad (24)$$

Using the empirical result that error in level truncation goes as $1/L$, we can fit the two data points (23, 24) through

$$c_4^{[n]} = \alpha + \beta/n \quad (25)$$

giving

$$\alpha \approx -1.686. \quad (26)$$

To three decimal places, the exact value for c_4 is

$$c_4^{\text{exact}} \approx -1.742. \quad (27)$$

Thus, the simple approximations given by truncating intermediate fields at levels 2 and 4 are able to reproduce the correct answer to within approximately 3%. The coefficient c_4 was computed in [9] using method (b), and in [18] using method (c) and oscillator truncation up to $L = 100$. While no closed-form expression for the result is known, numerical approximations in both cases give (27) with error of order 10^{-4} .

We have now described in some detail the calculation of the off-shell zero-momentum 4-tachyon amplitude in SFT. This calculation can be generalized in a straightforward fashion to compute all coefficients c_n in (15); the results of this calculation and their significance for the tachyon condensation problem are discussed in [1]. The calculation can also be generalized easily to include nonzero external momenta for the interacting tachyon fields, giving an off-shell generalization of the Veneziano amplitude. We now briefly review this generalization.

The on-shell 4-point tachyon amplitude for open bosonic strings is perhaps the best-known perturbative calculation in string theory [23]. To compute the on-shell amplitude at

tree level for four tachyons with momenta p_1, \dots, p_4 , one computes the disk amplitude with four tachyon insertions on the boundary. Three of the marked points on the boundary can be moved to canonical points through a conformal map from the disk to itself; the remaining marked point is a modular parameter which must be integrated over to compute the full 4-point amplitude. Often, the disk is mapped to the upper half-plane, and the fixed points are taken to be 0, 1, and ∞ , while the remaining point ξ is integrated over. The resulting on-shell Veneziano amplitude is given by

$$\mathcal{A}_4^{[V]}(p_1, p_2, p_3, p_4) = B(-s-1, -t-1) \quad (28)$$

where $B(u, v)$ is the Euler beta function, and s, t are the Mandelstam variables

$$s = -(p_1 + p_2)^2, \quad t = -(p_2 + p_3)^2, \quad (29)$$

The Euler beta function has the integral representation

$$B(u, v) = \int_0^1 d\xi \xi^{u-1} (1-\xi)^{v-1}. \quad (30)$$

This integral representation of the Veneziano amplitude is convergent when $u, v > 0$, so that $s, t < -1$. For positive real s, t it is necessary to perform an analytic continuation to make sense of (30) away from the poles at positive integers.

Using method (b) discussed in Section 2, Giddings reproduced the Veneziano amplitude from OSFT [7]. This calculation was generalized by Sloan and Samuel [12, 13] to off-shell momenta. They found that for momenta not satisfying $p^2 = 1$, the beta function integral (30) is replaced by

$$\int_0^1 d\xi \xi^{u-1} (1-\xi)^{v-1} \left(\frac{1}{2} \kappa(\xi) \right)^{\sum_i (p_i^2 - 1)}, \quad (31)$$

where $\kappa(\xi)$ is a complicated integral of functions defined implicitly by relations on elliptic integrals and the Jacobi zeta function. This function arises due to the complicated relationship between the modular parameter ξ and the Witten parameter x which appears in the OSFT calculation through (13).

An alternative approach to computing the off-shell 4-point tachyon amplitude for generic p is to use method (c) as above [13, 18]. Including the momenta, the calculation proceeds almost exactly as before, except that in the vertices and propagator, the exponents containing quadratic forms in the raising and lowering oscillators also contain terms of the form $ap, a^\dagger p$, and p^2 . The full amplitude can still be computed using squeezed state methods, and the result (14) takes the form

$$\frac{(\kappa g)^2}{2} \int_0^1 \frac{dx}{x^2} \frac{\det \left(\mathbb{1} + S(x) \cdot \tilde{X} \right)}{\det \left(\mathbb{1} - S(x) \cdot \tilde{V} \right)^{13}} e^{-\frac{1}{2} p_i Q^{ij}(x) p_j}, \quad (32)$$

where the quadratic form $Q^{ij}(t)$ can be expressed in terms of the Neumann coefficient matrices V_{nm}^{rs} . Using level truncation on oscillators to approximate this amplitude numerically

gives results which reproduce both the on-shell amplitude (28) and the off-shell amplitude (31) to high precision [18]. While this approach does not give an analytic result like method (b), it is much easier to generalize to more complicated diagrams.

4 Applications

In this section we briefly summarize two applications of perturbative computations in OSFT. First, we discuss the computation of the effective action for the massless vector field on a D-brane. Second, we discuss loop computations in OSFT.

4.1 Application: effective action for A_μ

In [1], we described the computation of the effective action for the zero-momentum tachyon field $S_{\text{eff}}(\phi)$. As discussed in the previous section, the coefficients c_n in this action can be computed by computing the n -point function for the tachyon and omitting the tachyon field on all internal propagators; this amounts to integrating out all other fields from the theory. In a similar way, we can compute an effective action for the massless vector field A_μ on a D-brane, by integrating out all the massive string fields, as well as the tachyon field. We could also consider integrating out the massive fields with $M^2 > 0$, but not the tachyon, to find an action for both ϕ and A_μ —we comment further on this possibility briefly below—but to compare with known on-shell scattering amplitudes of A_μ , it is perhaps most interesting to consider the effective action for A_μ alone.

The effective action $S_{\text{eff}}(A)$ which we compute in this fashion is essentially an off-shell low-energy action for a system of one (or many, if we include Chan-Paton factors) D-brane(s). One complication in computing this action is associated with the issue of gauge invariance. We would like an effective action which retains the usual gauge invariance $\delta A_\mu = \partial_\mu \lambda$ (and its nonabelian generalization). The method we described above for computing perturbative amplitudes, however, involved completely fixing gauge to Feynman-Siegel gauge. To retain some gauge invariance in the effective action, we choose not to fix the gauge associated with the field

$$\lambda(p)b_{-1}|0;p\rangle. \quad (33)$$

This leads to an extra space-time field associated with a state at level one,

$$\chi(p)b_{-1}c_0|0;p\rangle, \quad (34)$$

which must be included in the string field. In the low-energy action, χ becomes an auxiliary field which can be explicitly integrated out, giving a gauge-invariant action in terms of the vector field A_μ .

In the abelian case of a single D-brane, we know what to expect from this calculation. Because the on-shell condition for the vector field is $p^2 = 0$, we can perform a systematic double expansion of the action in p and A . The action should then take the form

$$S \sim \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})} + \text{derivative corrections} \quad (35)$$

where $F_{\mu\nu}$ is the abelian Yang-Mills field strength and the terms of order $F^n \sim \partial^n A^n$ organize into the Born-Infeld form, with terms of order $p^{n+k} A^n$, $k > 0$ considered as derivative corrections. In the nonabelian case, the situation is more subtle. Because derivatives can be exchanged with field strengths through relations of the form $[D, [D, F]] = [F, F]$, we cannot expand separately in A, p . Instead, all terms at order $p^k A^{n-k}$ should be considered together. In this case we expect that the action will take the “nonabelian Born-Infeld” form

$$S = -\frac{1}{4}\text{Tr } F^2 + \alpha_3 \text{Tr } F^3 + \alpha_4 \text{STr } \left(F^4 - \frac{1}{4}(F^2)^2 \right) + \dots \quad (36)$$

Much recent work has focused on computing the terms in this nonabelian action in the case of the superstring; for some recent results and further pointers to the literature see [24]. In the bosonic case, the action has been computed to order F^4 [25, 26, 27], and the coefficients α_3, α_4 are known to be $\alpha_3 = 2ig_{\text{YM}}/3, \alpha_4 = (2\pi g_{\text{YM}})^2/8$.

In [28], we computed all terms in the effective action for the massless field A to order $\partial^4 A^4$ using a finite level expansion and extrapolation in a power series in $1/L$ to compute all coefficients to 6 digits of accuracy. This computation correctly reproduces the Yang-Mills action both in the abelian and nonabelian theories. At finite level, the quartic term $(A^2)^2$ in the bosonic theory does not vanish, due to terms in the action of the form ϕA^2 . These terms, however, cancel to a high degree of accuracy when higher-level fields are included. It was shown analytically in [8] that this cancellation is exact—in that paper, the authors also carried out the analogous computation for the superstring, showing that the Yang-Mills action is also reproduced correctly in that theory.

The first novel result of the explicit calculation of the effective action for A is the appearance of nonzero terms at order $p^2 A^4$ in the abelian theory. Such terms do not appear in the Born-Infeld action (35). The explanation for this discrepancy is rather illuminating. In the full SFT, the transformation rule for A_μ contains not only the term $\partial_\mu \lambda$ linear in the gauge parameter appearing in (33), but also terms which are linear in λ and proportional to the massive string fields. When the massive fields are integrated out, the full transformation rule becomes

$$\delta A_\mu = \partial_\mu \lambda + \mathcal{O}(\lambda A^2). \quad (37)$$

In order to relate the effective field transforming in this way to the usual gauge field \hat{A}_μ transforming only under $\partial_\mu \lambda$, it is necessary to perform a field redefinition

$$\hat{A}_\mu = A_\mu + \gamma_1 A^2 A_\mu + \gamma_2 A^2 \partial^2 A_\mu + \dots \quad (38)$$

Field redefinitions of this type which relate SFT fields to the usual space-time fields associated with CFT states were discussed in [29, 30]. It was shown in [28] that after such a field redefinition, the F^3 and F^4 terms in the nonabelian and abelian Born-Infeld actions are correctly reproduced from SFT. Some comments regarding this computation may be helpful:

- It is important to emphasize that the “natural” Yang-Mills variables appropriate for describing physics on a D-brane are not at all apparent in SFT. To get to the natural fields for the D-brane background, it is necessary to do a field redefinition. On

the one hand, this makes the SFT formulation of the theory in the D-brane background seem quite obscure. On the other hand, this complication is a very natural feature of a background-independent theory. In any background-independent theory, one should expect to have to do a complicated field redefinition to go from the degrees of freedom associated with one background to the degrees of freedom associated with another background. We see here that even though we have chosen a particular background around which to expand the theory, SFT does not really use the natural fields associated with that background. This highlights the complexity of describing physics in distinct backgrounds within the framework of SFT or any other background-independent theory. The gravitational analogue of this observation is that in closed string field theory, or any other formulation of string theory which is independent of closed string backgrounds, we might expect that to write the usual metric degrees of freedom associated with a particular background in terms of the closed string fields, a field redefinition of the type

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \alpha g_{\mu\lambda} g_{\nu}^{\lambda} + \cdots \quad (39)$$

would be necessary

- Another point worth noting is that the appearance of derivatives in the field redefinition (38) has important implications for the geometrical interpretation of the effective theory. In the usual Born-Infeld action, the transverse scalar fields X^i encode the transverse positions of the D-branes. On a D-brane of nonzero codimension, these fields are related through T-duality to the gauge field components A_i on a higher-dimensional D-brane, and will transform under the same type of transformation rule as (38). Two D-branes with identical values of the scalars \hat{X}^i at a particular point ξ in the D-brane world-volume are coincident at that point, and have a massless string mode attaching them. On the other hand, after a field redefinition of the form (38), this geometry becomes much more obscure, as the redefined fields X^i need *not* be the same even at a point of coincidence. Thus, in the SFT formulation, the notion of locality on D-branes is rather subtly encoded in the fields of the theory.
- The calculation described above could also be generalized to compute an effective action $S_{\text{eff}}[\phi, A]$ for both the tachyon field ϕ and the gauge field A_μ , as mentioned above, by only integrating out fields with positive mass squared. Because in this case, however, the on-shell equation for the tachyon is $p^2 = 1$ and not $p^2 = 0$, all derivative corrections will a priori be equally important, and there is no systematic expansion of this action in p, ϕ , and A . Nonetheless, it may be that one can learn something of the physics of this theory by including terms to all orders in p .

4.2 Application: Loops in OSFT

As a final application of the methods described in Section 2, we consider the computation of loop amplitudes. Computing loop amplitudes is necessary in order to ascertain whether

OSFT is truly a sensible quantum theory. An extended discussion of these issues is presented in the review article of Thorn [31]. The simplest loop diagrams are the annulus diagrams with one or two open string vertices on the boundaries. The one-loop two-point diagram was computed by Freedman, Giddings, Shapiro and Thorn in [32], and the one-loop one-point function was computed in [33].

In [32], the one-loop two-point diagram was computed using method (b) of Section 2. These authors found good evidence that this diagram has closed string poles, as expected. Thus, if OSFT is to be a good (unitary) quantum theory, it must in some way include closed string degrees of freedom as asymptotic states.

In [33], we considered the one-loop open string tadpole diagram, which we computed both using method (b) and method (c). Using both methods, it is possible to isolate the divergences coming from closed strings. Using method (c), the diagram can simply be written as

$$\langle T| = \int d^{26}q \int dt \langle V_3|_{123} e^{-tL_0^{(3)}} |V_2\rangle_{23}, \quad (40)$$

where q is the internal (loop) momentum. This state in \mathcal{H}^* gives the tadpole for any field in the theory by contracting with the associated state in \mathcal{H} . It can be thought of as giving rise to a linear term in the effective action $\langle T|\Phi\rangle$ where $|\Phi\rangle$ is the string field. In terms of squeezed states, the tadpole $\langle T|$ can be written as

$$\langle T| = \int_0^\infty dt e^t \langle 0|c_0 \exp\left(-\frac{1}{2}a \cdot M(t) \cdot a - c \cdot R(t) \cdot b\right) \frac{\det(\mathbb{1} - S(t) \cdot \tilde{X})}{\left(Q(t) \det(\mathbb{1} - S(t) \cdot \tilde{V})\right)^{13}}, \quad (41)$$

where $Q(t)$ is a scalar function of t and $S, \tilde{X}, \tilde{V}, M, R$ are infinite matrices defined appropriately for the one-loop tadpole diagram. (The detailed form of these matrices is given in [33]). This integral diverges as $t \rightarrow 0$. The appearance of this divergence is clear in the closed string picture. As $t \rightarrow 0$, the annulus becomes a long, thin cylinder, which in the closed string picture represents a closed string propagating along a world-sheet of length $s = 1/t \rightarrow \infty$. The closed string tachyon leads to a divergence, as the closed string propagator takes the form

$$\int ds e^{-sM^2}, \quad (42)$$

which diverges when $M^2 < 0$ and $s \rightarrow \infty$. This divergence can be seen both in a computation using method (b) and in (41). In both cases, the form of the state $\langle T|$ as $t \rightarrow 0$ corresponds to a Shapiro-Thorn state [34] representing the closed string tachyon.

One very promising feature of this loop calculation is that we see explicitly how the closed strings play a fundamental role in the open string loop calculation. The off-shell open string tadpole arises essentially from the fact that in a D-brane background, the closed strings acquire tadpoles associated with the linearized solution of the supergravity equations in the D-brane background. Because of the coupling between open and closed strings, the closed string background manifests as a nontrivial open string tadpole. While this shows that closed strings indeed arise naturally in OSFT, the difficulties associated with the closed

string tachyon seem to indicate that it is unlikely that a complete and consistent formulation of the quantum bosonic OSFT exists.

The tadpole (41) has in fact several types of divergence. There is a fairly harmless open string divergence like that of (14) associated with the open string tachyon propagating around the loop in the limit $t \rightarrow \infty$. This divergence is easy to analytically continue or remove by hand. The divergence mentioned above which is associated with the closed string tachyon, however, is much more difficult to deal with systematically. Removing this divergence seems to require that we “cheat” by using our knowledge of the closed string structure to analytically continue the closed string tachyon. This resolution is difficult to imagine continuing to general loop order since the closed strings are not really fundamental degrees of freedom in OSFT, so each diagram must essentially be treated by hand, making any possibility of nonperturbative results rather unlikely. These problems seem to indicate that, in the absence of some new and as-yet-unsuspected magic, the quantum bosonic OSFT is probably not a well-defined unitary theory, and that to have a complete quantum string field theory one must probably go to the superstring. Indeed, there is as yet no reason to believe that any of the problems encountered here should afflict the superstring. Several candidates for a classical superstring field theory exist, including the Berkovits formalism [35], as well as possibly a variation of the cubic Witten theory [36, 37, 38, 39, 40, 41]— for a recent comparison of these approaches see [42]. A good classical superstring field theory should lead naturally to a sensible quantum superstring field theory, at least in a background associated with a Dp -brane with $p < 7$. For $p \geq 7$, there are infrared divergences, also seen in the bosonic theory, which complicate the problem somewhat; these divergences are simply those which arise from the linearized gravity equations in the presence of a source, and presumably do not represent a fundamental problem with the theory. It seems quite plausible that superstring field theory in a Dp -brane background can be formulated as a completely well-defined and sensible nonperturbative quantum theory.

5 Conclusions

We have described several ways in which string field theory can be used to compute on-shell and off-shell perturbative amplitudes. String field theory offers a simple and consistent way to compute arbitrary diagrams, including in principle even diagrams at very high loop order. On the other hand, at this point the methods available for computing diagrams in SFT are primarily based on numerical approximation methods. These methods involve the truncation of an infinite family of terms to a finite subset and an extrapolation by matching to a simple power series in the reciprocal of the cutoff.

While we still do not have any completely rigorous demonstration of the effectiveness of the level truncation method, empirical evidence indicates that it converges well for a wide class of diagrams. At some point in the future it would be nice to develop a more systematic approach to this approximation scheme, so that errors can be more accurately estimated. Still, even using fairly basic methods, we have been able to calculate useful amplitudes to 5

or 6 digits of precision using the level cutoff scheme.

In principle, OSFT coupled with level truncation can be used to compute arbitrary string amplitudes to arbitrary precision. For the bosonic theory, however, loop amplitudes are plagued with various divergences. Among these, the divergences from the closed string tachyon seem to render the theory sufficiently unstable that it is difficult to imagine extracting physically useful information from bosonic loop amplitudes, let alone nonperturbative quantum effects in the bosonic theory. To begin to address questions relevant to a physical theory, it is probably necessary to have a consistent formulation of superstring field theory. Berkovits [35] has made some progress in this direction, and initial perturbative results in the theory such as that of [8] are quite promising, but the quantum features of superstring field theory remain to be investigated.

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